British Astronomy and Astrophysics Olympiad 2017-2018

Astronomy & Astrophysics Challenge Paper

September - December 2017

Instructions

Time: 1 hour.

Questions: Answer all questions in Sections A and B, but only **one** question in Section C.

Marks: Marks allocated for each question are shown in brackets on the right.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets. Students should ensure their name and school is clearly written on all answer sheets and pages are numbered. A standard formula booklet with standard physical constants should be supplied.

Eligibility: All sixth form students (or younger) are eligible to sit this paper (and any others in this year's BAAO competition).

Further Information about the British Astronomy and Astrophysics Olympiad

This is the first paper of the British Astronomy and Astrophysics Olympiad in the 2017-2018 academic year. To progress to the next stage of the BAAO, you **must** *take the BPhO Round 1 in November 2017, which is a general physics problem paper. Those achieving at least a Gold will be invited to take the BAAO Competition paper on* **Monday 22nd January 2018**.

To be awarded the highest grade (Distinction) in this paper, it should be sat under test conditions and marked papers achieving 60% or above should be sent in to the BPhO Office in Oxford by **Friday 20th October 2017**. *Papers sat after that date, or below that mark (i.e. Merit or Participation), should have their results recorded using the online form by* **Friday 8th December 2017**.

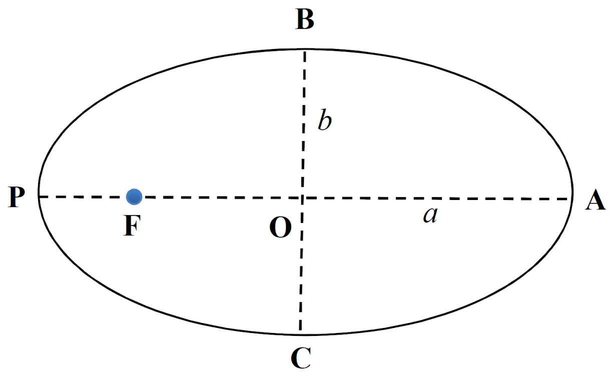
To solve some of the questions, you will need to write equations, draw diagrams and, in general, show your working. You are also encouraged to look at the clear sky and identify the brightest stars, a few days before sitting the paper.

This paper has more than an hour's worth of questions. You are encouraged to have a go at as many as you can and to follow up on those that you do not complete in the time allocated.

Important Constants

Constant	Symbol	Value
Speed of light	c	$3.00 \times 10^8\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	AU	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Earth	$\mathrm{R_E}$	$6.37 \times 10^6\,\mathrm{m}$
Semi-major axis of the Earth's orbit		1 AU
Radius of the Sun	$\mathrm{R_\odot}$	$6.96 \times 10^8\,\mathrm{m}$
Mass of the Sun	$\mathrm{M_\odot}$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	$\mathrm{M_E}$	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$\mathrm{L_\odot}$	$3.85 \times 10^{26}\,\mathrm{W}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

$a = \mathbf{OA}\ (= \mathbf{OP})$	semi-major axis
$a = \mathbf{OB}\ (= \mathbf{OC})$	semi-minor axis
$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
F	focus
P	periapsis (point nearest to F)
A	apoapsis (point furthest from F)

Keplers Third Law: For an elliptical orbit, the square of the period, T, of orbit of an object about the focus is proportional to the cube of the semi-major axis, a (the average of the minimum and maximum distances from the Sun). The constant of proportionality is $4\pi^2/GM$, where M is the mass of the central object and G is the universal gravitational constant.

Magnitudes: The apparent magnitudes of two objects, m_1 and m_0, are related to their apparent brightnesses, b_1 and b_0, via the formula:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

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Section A: Multiple Choice

Write the correct answer to each question. Each question is worth 1 mark. There is only one correct answer to each question. **Total: 10 marks**.

1. In 2015, physicists at the Laser Interferometer Gravitational-Wave Observatory (LIGO) announced the first-ever observation of gravitational waves. What triggered the pulse?

   A. Two neutron stars colliding

   B. Two black holes colliding

   C. A star going supernova

   D. A gamma ray burst

2. In which constellation would you find the centre of the Milky Way?

   A. Ophiucus

   B. Coma Berenices

   C. Sagittarius

   D. Scorpius

3. What is the approximate average thickness of Saturn's rings?

   A. 10 μm

   B. 10 mm

   C. 10 m

   D. 10 km

4. Pluto has roughly the same surface area as which country?

   A. Australia

   B. China

   C. USA

   D. Russia

5. In the video game *Halo*, players are able to move around on the inner edge of a ring structure, which has a radius of 5000 km. Roughly how many rotations per (Earth) day would the ring need to achieve in order for the players' characters to experience Earth-like gravity whilst walking around?

   A. 15

   B. 17

   C. 19

   D. 21

6. What is the declination of the Sun on the Winter solstice?

   A. $+45°$

   B. $+23.5°$

   C. $0°$

   D. $-23.5°$

7. Which of the following cities will experience the longest day during June?

   A. Edinburgh (longitude $= 3.2°$W, latitude $= 56.0°$N)
   B. Rome (longitude $= 12.5°$E, latitude $= 41.9°$N)
   C. Nairobi (longitude $= 36.8°$E, latitude $= 1.3°$S)
   D. Sydney (longitude $= 58.4°$W, latitude $= 34.6°$S)

8. If a planet orbits the Sun with a semi-major axis of 4 AU, what is the period of its orbit?

   A. 4 years
   B. 8 years
   C. 12 years
   D. 64 years

9. Two light sources, A and B, emitting their light isotropically (i.e. equally in all directions) are placed at distance r and $2r$ respectively from a detector, which shows they have the same apparent brightness (i.e. $b_A/b_B = 1$). If A is moved to $2r$ and B is moved to $3r$, what is the new ratio of apparent brightness b'_A/b'_B?

   A. $2/3$
   B. $4/9$
   C. $3/4$
   D. $9/16$

10. Light source A from the previous question is returned to a distance of r from the detector. How far away should it now be moved to appear 5 magnitudes fainter?

   A. $5r$
   B. $10r$
   C. $50r$
   D. $100r$

Section B: Short Answer

Each short question is worth 5 marks. **Total: 10 marks**.

Day of the Equinox

11. The word 'equinox' comes from the Latin for "equal night", and typically people assume it's the time of year when day and night are equal in length (i.e. 12 hours each), but that's not strictly true - in fact the centre of the solar disc spends 12 hours above the horizon (see Fig 1), but sunrise and sunset times are measured from when the very top of the Sun first appears, and when it finally disappears respectively.

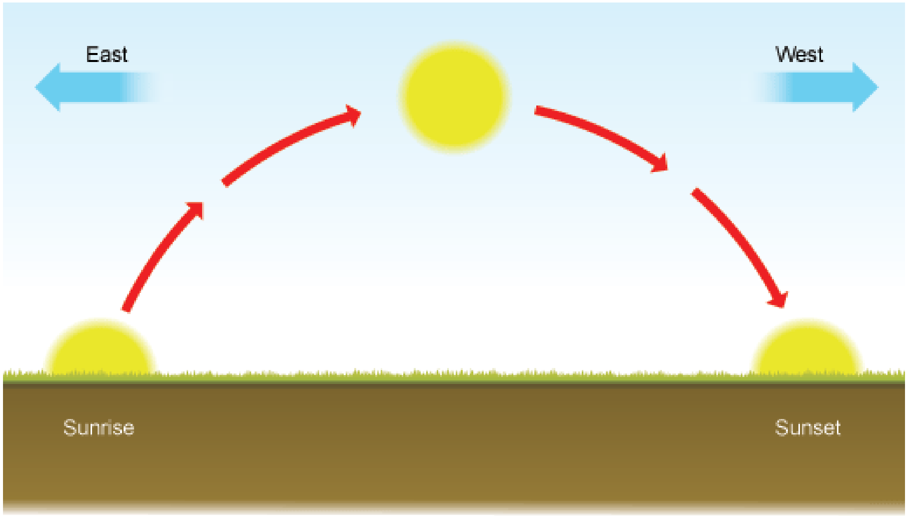

Figure 1: The motion of the Sun during the 12 hours of an equinox.

(a) By working out the angular radius of the Sun as seen from the Earth, work out how many extra minutes of daylight you get from this effect.

[3]

(b) Since the atmosphere has a refractive index just slightly greater than 1, this has an influence too, meaning that when the centre of the solar disc appears to be just at the horizon, it is in fact 0.6° below the horizon. Combining this with the previous effect, calculate the total number of minutes longer than 12 hours a day is on an equinox.

[2]

Stellar Collapse

12. Star formation begins when dense clumps within giant molecular clouds, called diffuse nebulae, begin to collapse as their gravity exceeds the pressure from the temperature of the gas. For much of the collapse the clump remains transparent and so can radiate away the energy of the collapse. Consequently the clump stays at a very constant temperature (until a protostar begins to form at the core, when the temperature does rise rapidly as the gas becomes opaque), and so this part of the collapse happens essentially in freefall. This means it can happen rather fast (by astronomical timescales!).

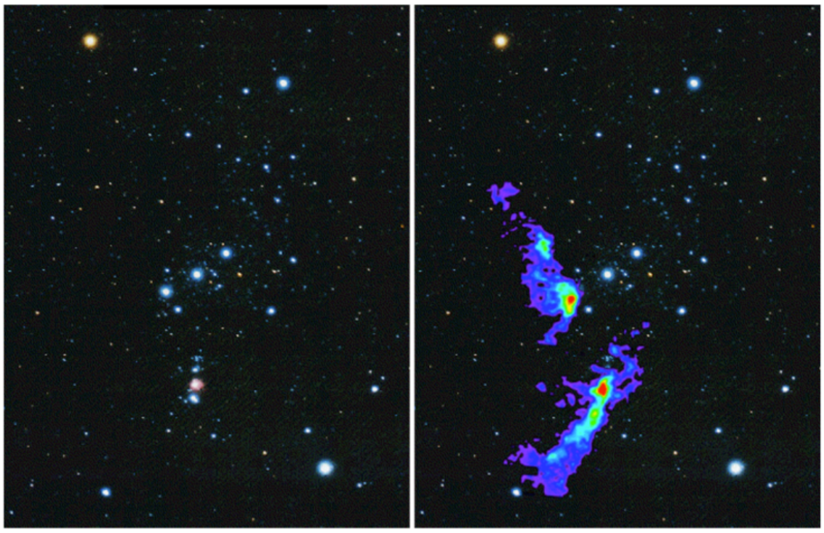

Figure 2: *Left*: Orion in visible light.
Right: With radio detections of giant molecular clouds superimposed.

Consider one of the very dense clumps within a nebula. Assume it is spherical with an initial radius of r_0, and begins to collapse at time $t = 0$, until it has shrunk to radius r by some time t later. If the initial density of this clump, ρ_0, is uniform everywhere in the sphere (called a homologous collapse) then we can describe its collapse with the following formulae:

$$\theta + \frac{1}{2}\sin 2\theta = \left(\frac{8\pi G\rho_0}{3}\right)^{1/2} t \qquad \text{where} \qquad \frac{r}{r_0} = \cos^2\theta$$

(a) The duration of the freefall is t_{ff}, and by the end of that part of the collapse $r \ll r_0$. Using the above equations derive an expression for t_{ff} in its simplest form. What do you notice about your formula? [Note: θ is in radians]

[3]

(b) If the density of the clump is 5.0×10^{-16} kg m^{-3}, calculate a value for t_{ff}. Give you answer in years.

[1]

(c) Draw what you expect a graph of r against t to look like.

[1]

Section C: Long Answer

Each long question is worth 10 marks. Answer **either** *Qu 13* **or** *Qu 14.* **Total: 10 marks**.

Solar Wind

13. From the upper parts of the Sun's corona comes a stream of charged particles called the solar wind, meaning the Sun is slowly losing some of its mass (although the effect is negligible compared to the mass loss in nuclear fusion). The particles travel at supersonic speeds until the pressure from interstellar space causes their speed to drop to subsonic speeds instead - this transition is called the termination shock, and has been explored by the two Voyager probes as they leave the solar system.

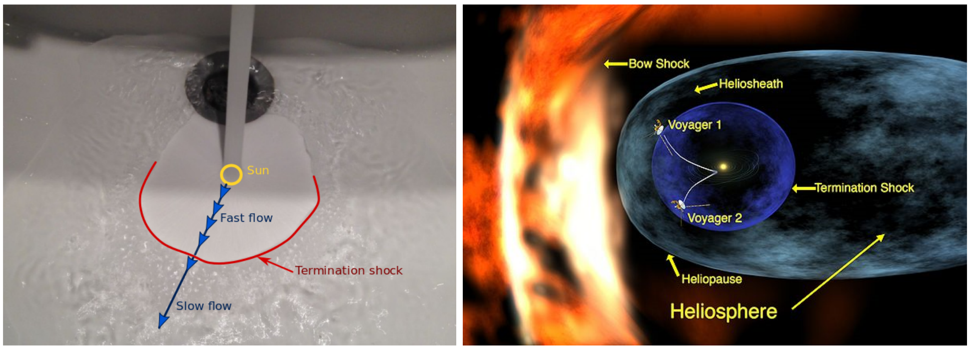

Figure 3: *Left*: A demonstration of a termination shock formed with water flowing from a tap into a sink. *Right*: The Voyager spacecraft crossing the termination shock of the Solar System.

(a) By considering the solar wind to be radiated equally in all directions as a spherical shell, derive a formula for the rate of mass lost by the Sun, $\Delta M/\Delta t$, given the density, ρ, and speed, v, of the solar wind at a given distance r.

[2]

(b) At the orbit of the Earth, the solar wind is measured to have a density of 7 protons cm^{-3} and to be travelling at 500 km s^{-1}. Calculate $\Delta M/\Delta t$, giving your answer in units of $\mathrm{M}_\odot$ year^{-1}.

[2]

(c) As the solar wind moves further from the Sun its speed increases (at an ever decreasing rate), until it asymptotes at a speed, u_∞, equal to the escape velocity, v_{esc}, of the Sun. Given that $v_{\text{esc}} = \sqrt{2GM/R}$ for an object with radius R and mass M, calculate the escape velocity of the Sun. Give your answer in km s^{-1}.

[1]

(d) At the termination shock boundary the particles slow down from their (supersonic) u_∞ to a much slower (subsonic) speed. Another way of defining this boundary is that it is where the pressure of the interstellar medium, P_{ISM}, is equal to the kinetic energy density (i.e. KE per unit volume) of the gas. If P_{ISM} is estimated to be 10^{-13} Pa, use your values of u_∞ and $\Delta M/\Delta t$ to calculate an estimate for the distance to the termination shock. Give your answer in AU.

[4]

(e) Voyager 1 detected the termination shock at a distance of 94 AU, whilst Voyager 2 detected it at a distance of 84 AU. What does this tell us about the shape of the boundary?

[1]

Double Planet

14. When two objects of unequal mass orbit around each other, they both orbit around a barycentre - this is the name given to the location of the centre of mass of the system. The masses of both objects, and the distance between their centres, affects the position of their barycentre. Imagine two objects, Object 1 and Object 2, with masses m_1 and m_2 respectively, and the average distance between the centre of both objects is a, then the average distance from the centre of Object 1 to the barycentre, r, is given by the formula:

$$r = a\frac{m_2}{m_1 + m_2}$$

(a) Verify that the barycentre of the Sun-Jupiter system lies outside the Sun (i.e. $r > \mathrm{R}_\odot$), given that the mass of Jupiter is 1.90×10^{27} kg and it is on average 5.20 AU from the Sun.

[2]

Figure 4: The relative positions of Pluto, Charon and the system's barycentre.

(b) One of the most famous examples of a system with the barycentre lying outside the larger object is the Pluto-Charon system (see Fig 4), which is why many considered it to be a double-planet long before it was reclassified as a dwarf planet. Given the barycentre of the system is at $1.83\ \mathrm{R}_{\mathrm{Pluto}}$ (where $\mathrm{R}_{\mathrm{Pluto}} = 1187$ km) and the average separation between Pluto and Charon (as measured from centre to centre) is 19 570 km, calculate the ratio of their masses (i.e. $m_{\mathrm{Pluto}}/m_{\mathrm{Charon}}$).

[3]

(c) Some have claimed that a double planet should be distinguished from a planet and large moon when a system fulfils the criterion $r > R_1$. The Earth-Moon system does not currently satisfy that condition for a double planet despite the Moon being rather large relative to the Earth, but the Moon is slowly moving away from the Earth at roughly 4 cm per year. Assuming this rate stays constant, calculate the number of years until $r = \mathrm{R}_{\mathrm{E}}$. [Average distance between centres of Earth and Moon = 384 400 km, and the Earth has 83.1 times the mass of the Moon.]

[3]

(d) The Moon's orbit is not quite circular - it is in fact an ellipse with an eccentricity of 0.055. Will this mean your answer to part (c) is an overestimate or an underestimate? Explain your reasoning.

[2]

END OF PAPER

Questions proposed by:
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