British Astronomy and Astrophysics Olympiad 2019-2020

Astronomy & Astrophysics Challenge Paper

September - December 2019

Instructions

Time: 1 hour (30 marks).

Questions: Answer all questions in Sections A and B, but only **one** question in Section C.

Marks: Marks allocated for each question are shown in brackets on the right. Working must be shown in order to get full credit, and you may find it useful to write down numerical values of any intermediate steps.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets. Students should ensure their name and school is clearly written on all answer sheets and pages are numbered. A standard formula booklet with standard physical constants should be supplied.

Eligibility: All sixth form students (or younger) are eligible to sit any BAAO paper.

Further Information about the British Astronomy and Astrophysics Olympiad

This is the first paper of the British Astronomy and Astrophysics Olympiad in the 2019-2020 academic year. To progress to the next stage of the BAAO, you **must** *take the BPhO Round 1 in November 2019, which is a general physics problem paper. Those achieving at least a Gold will be invited to take the BAAO Competition paper on* **Monday 20th January 2020**.

To be awarded the highest grade (Distinction) in this paper, it should be sat under test conditions and marked papers achieving 60% or above should be sent in to the BPhO Office in Oxford by **Friday 18th October 2019**. *All papers, including papers sat after that date, or below that mark (i.e. Merit or Participation), should have their results recorded using the online form by* **Friday 6th December 2019**.

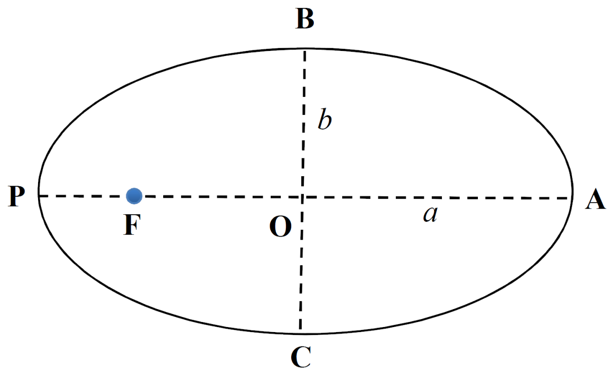
To solve some of the questions, you will need to write equations, draw diagrams and, in general, show your working. You are also encouraged to look at the clear sky and identify the brightest stars, a few days before sitting the paper.

This paper has more than an hour's worth of questions. You are encouraged to have a go at as many as you can and to follow up on those that you do not complete in the time allocated.

Important Constants

Constant	Symbol	Value
Speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	au	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Earth	R_E	$6.37 \times 10^{6}\,\mathrm{m}$
Semi-major axis of the Earth's orbit		1 au
Radius of the Sun	$R_{\odot}$	$6.96 \times 10^{8}\,\mathrm{m}$
Mass of the Sun	$M_{\odot}$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	M_E	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$L_{\odot}$	$3.85 \times 10^{26}\,\mathrm{W}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

	$a = \mathbf{OA}\ (= \mathbf{OP})$	semi-major axis
	$b = \mathbf{OB}\ (= \mathbf{OC})$	semi-minor axis
	$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
	F	focus
	P	periapsis (point nearest to F)
	A	apoapsis (point furthest from F)

Kepler's Third Law: For an elliptical orbit, the square of the period, T, of an object about the focus is proportional to the cube of the semi-major axis, a (as defined above), such that

$$T^2 = \frac{4\pi^2}{GM} a^3 ,$$

where M is the total mass of the system (typically dominated by the central object) and G is the universal gravitational constant.

Magnitudes: The apparent magnitudes of two objects, m_1 and m_0, are related to their apparent brightnesses, b_1 and b_0, via the formula:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

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Section A: Multiple Choice

Write the correct answer to each question. Each question is worth 1 mark. There is only one correct answer to each question. **Total: 10 marks**.

1. The image below was released in April 2019 by the Event Horizon Telescope collaboration and is considered to be one of the most significant astronomical images ever made. What is it of?

   A. A supernova
   B. A planetary nebula
   C. A black hole
   D. A quasar

2. Where on the Moon did astronauts walk in July 1969?

   A. Mare Serenitatis
   B. Mare Tranquillitatis
   C. Mare Imbrium
   D. Mare Nubium

3. The Chandra X-ray Observatory celebrates 20 years of operation this year, and was 100 times more sensitive than previous X-ray telescopes when it was launched. All X-ray telescopes have either been space-borne or operate in near-space environments. This is because:

   A. X-rays cannot penetrate the Earth's atmosphere all the way to the ground
   B. on the ground there is too much interference from medical X-rays
   C. it is dangerous to be close to an X-ray telescope so it must be highly remote from human life
   D. the resolution of the telescope would be too poor for astronomical observations from the ground

4. Looking up into the UK night sky in late September, which of the following bright stars rises first?

   A. Aldebaran (Right ascension $= 04^{\mathrm{h}}\ 36^{\mathrm{m}}$, declination $= +16.51°$)
   B. Rigel (Right ascension $= 05^{\mathrm{h}}\ 15^{\mathrm{m}}$, declination $= -8.20°$)
   C. Procyon (Right ascension $= 07^{\mathrm{h}}\ 39^{\mathrm{m}}$, declination $= +5.23°$)
   D. Sirius (Right ascension $= 06^{\mathrm{h}}\ 45^{\mathrm{m}}$, declination $= -16.72°$)

5. In May 1919 the British astronomers Frank Watson Dyson and Arthur Stanley Eddington organised for two teams to photograph a total solar eclipse, one from the West African island of Principe and the other from the Brazilian town of Sobral. Which aspect of a new scientific theory were their observations seen as decisive evidence for?

   A. The bending of light close to an object as predicted by Einstein's general relativity
   B. The existence of quantised electron energy levels as predicted by Bohr's model of the atom
   C. The Sun is undergoing nuclear fusion in its core as predicted by Eddington
   D. The solar corona has a strong magnetic field as predicted by Maxwell's equations

6. An observer in the UK sees a full moon in the constellation of Capricorn. What time of year is it?

   A. Spring
   B. Summer
   C. Autumn
   D. Winter

7. Earth's moon has a radius of 1737 km and Titan (one of Saturn's moons) has a radius of 2576 km. At the surface, their gravitational field strengths, g, are 1.63 N kg^{-1} and 1.35 N kg^{-1} respectively. Determine the ratio of masses, $M_{\text{Titan}}/M_{\text{Moon}}$, given $g \propto M/R^2$.

   A. 1.23
   B. 1.79
   C. 1.82
   D. 2.66

8. The One-Mile Telescope is a radio telescope just outside Cambridge that has three dishes that can be spread out up to one mile ($= 1.6$ km) apart. Two of the dishes are fixed, whilst one can move along an 800-m set of former railway tracks. In order for the tracks to be perfectly flat, how much did they need to raise one end to compensate for the curvature of the Earth?

   A. 5 cm
   B. 10 cm
   C. 15 cm
   D. 20 cm

9. There are approximately 450 geostationary satellites currently in orbit. They have an orbital period of 1 day and orbit in the plane of the equator, so are therefore always directly above the same spot. Assuming circular orbits and an equidistant arrangement, what is the distance between two geostationary satellites?

   A. 560 km
   B. 570 km
   C. 580 km
   D. 590 km

10. On the 1st January 2019 the New Horizons probe (having successfully flown by Pluto in 2015) had an encounter with the Kuiper belt object Ultima Thule, making it the most distant object ever visited by a spacecraft. At the time it was 43.4 au from the Sun. Given the apparent magnitude of the Sun from the Earth is -26.74, what is the apparent magnitude of the Sun from Ultima Thule?

    A. -18.25
    B. -18.35
    C. -18.45
    D. -18.55

Section B: Short Answer

Each short question is worth 5 marks. **Total: 10 marks**.

Mercurian Days

11. Mercury is the innermost of the Solar System's planets and so is most influenced by gravitational interactions with the Sun, tying its rotational period to its orbital period in a similar way to the tidal locking between the Moon and the Earth. It orbits the Sun in a 3:2 resonance, meaning that it rotates on its axis three times for every two orbits of the Sun.

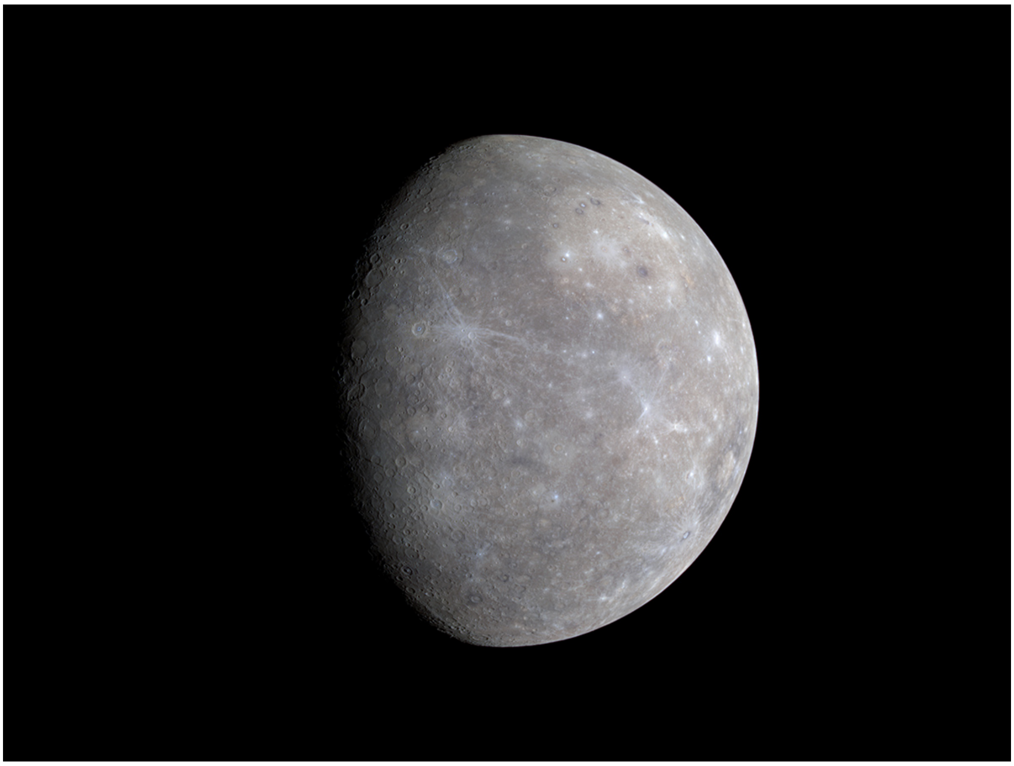

Figure 1: True colour image of Mercury taken by the probe MESSENGER after its closest approach in 2008. Credit: NASA/Johns Hopkins University Applied Physics Laboratory/Carnegie.

Mercury has an orbital period of 88 Earth days and a radius of 2440 km, and spins in the same direction as it orbits (both are anti-clockwise as viewed from high above the Sun).

(a) Thinking carefully about the geometry of the situation, calculate the length of a solar day as observed on Mercury (i.e. the length of time from one noon to the next). *Hint:* you should find it is longer than a Mercurian year.

[2]

(b) Hence calculate the speed an astronaut would need to move at whilst travelling along its equator in order to keep the Sun in the same position in the sky. Assume Mercury has no axial tilt, and give your answer in km h^{-1}.

[3]

Age of the Universe

12. A consequence of the expansion of the Universe is that galaxies appear to be moving away from us, and the further they are away the quicker they seem to be moving. Edwin Hubble showed that if the expansion was uniform in all directions then the relationship can be expressed as $v = H_0 d$ where v is the recessional velocity, d is the distance to the galaxy, and H_0 is called the Hubble constant. His original compilation of this data from his 1929 paper is shown in Figure 2.

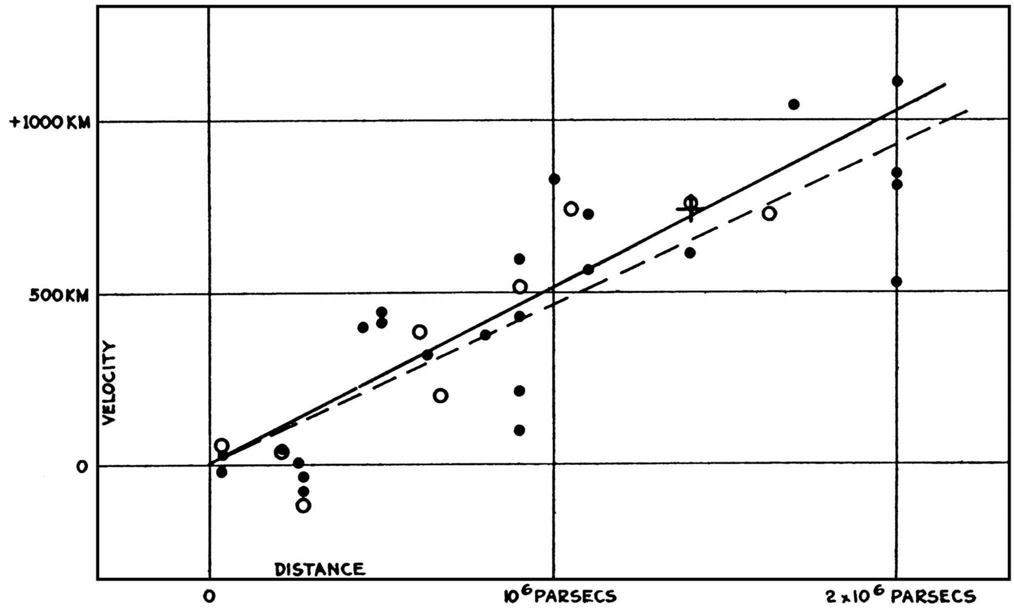

Figure 2: The original plot of recessional velocity (based upon redshifts) given in km s^{-1} (despite the incorrect axes labels) and distances given in Mpc (= 10^6 pc) to 32 nearby galaxies. The solid line shows the best fit to the individual data points. Credit: Hubble (1929).

Whilst the measured redshifts used to derive the values of v are largely consistent with the modern values, the distances are considerably different from the ones we accept today. Even so, we can repeat the analysis he did to get a rough value for the age of the Universe, despite it being rather different from our current estimates.

(a) Making full use of the graph, calculate the value of H_0, giving your answer in units of km s^{-1} Mpc^{-1}.

[2]

(b) The age of the Universe, t, is related to H_0 by $t = kH_0^n$ where k is a unitless numerical factor. By considering the dimensions of H_0 in SI units, find a value for n and, given $k = 1$, calculate t from your value of H_0. Give your answer in Gyr (= 10^9 years).

[3]

Section C: Long Answer

Each long question is worth 10 marks. Answer **either** *Qu 13* **or** *Qu 14.* **Total: 10 marks**.

Stellar Nuclear Fusion

13. Main sequence stars fuse hydrogen atoms to form helium in their cores. About 90% of the stars in the Universe, including the Sun, are main sequence stars. These stars can range from about a tenth of the mass of the Sun to up to 200 times as massive. The main source of energy in main sequence stars is from nuclear fusion. The mass of one hydrogen nucleus is $m_{\rm H} = 1.674 \times 10^{-27}$ kg, and the mass of one helium nucleus is $m_{\rm He} = 6.649 \times 10^{-27}$ kg.

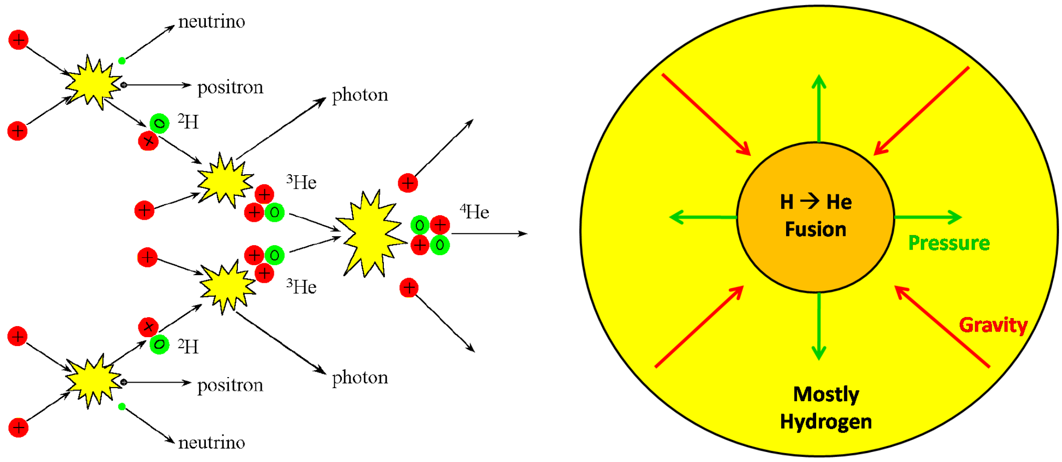

Figure 3: *Left*: The proton–proton chain reaction is one of two known sets of nuclear fusion reactions by which stars convert hydrogen to helium. It dominates in stars with masses less than or equal to that of the Sun's, and involves a net change of four hydrogen nuclei becoming one helium nucleus.
Right: Only the core of a main sequence star will undergo nuclear fusion due to the higher temperature than the surrounding hydrogen shell.

(a) By considering the difference in mass between the net inputs and outputs in the proton-proton chain, how much energy is produced in this reaction? *Hint*: you will need to use Einstein's most famous equation.

[1]

(b) Calculate the number of hydrogen nuclei that must fuse to form helium every second to provide the measured solar luminosity, and the percentage of hydrogen mass per reaction that is converted into energy via fusion.

[2]

(c) The Sun is composed of about 71% hydrogen, 27% helium and some heavier elements. However, only 13% of the hydrogen is available for hydrogen fusion in the core. The rest remains in layers of the Sun where the temperature is too low for fusion to occur. Use these figures and the answers to previous questions to calculate the lifetime in years of hydrogen fusion in the Sun (called its main sequence lifetime), assuming the Sun's luminosity remains constant.

[3]

(d) For main sequence stars the luminosity scales with the mass as $L \propto M^{3.5}$. Consider a binary star system with two stars separated by 9.1 au and an orbital period of 3.83 years. If the stars are unequal in mass, with one being three times heavier than the other, and assuming similar structures to the Sun, estimate the main sequence lifetime of the larger star.

[4]

Life in the TRAPPIST-1 System

14. In February 2017 the astronomical world was very excited by the announcement of the discovery of seven exoplanets similar in size to Earth orbiting around the ultra-cool red dwarf star TRAPPIST-1, found only 12.14 pc away. Whilst they are much closer to their star than Mercury is to the Sun, since the star is considerably cooler it means that three of them (planets d, e and f) are in the habitable zone of the star (defined as where liquid water could exist on the surface).

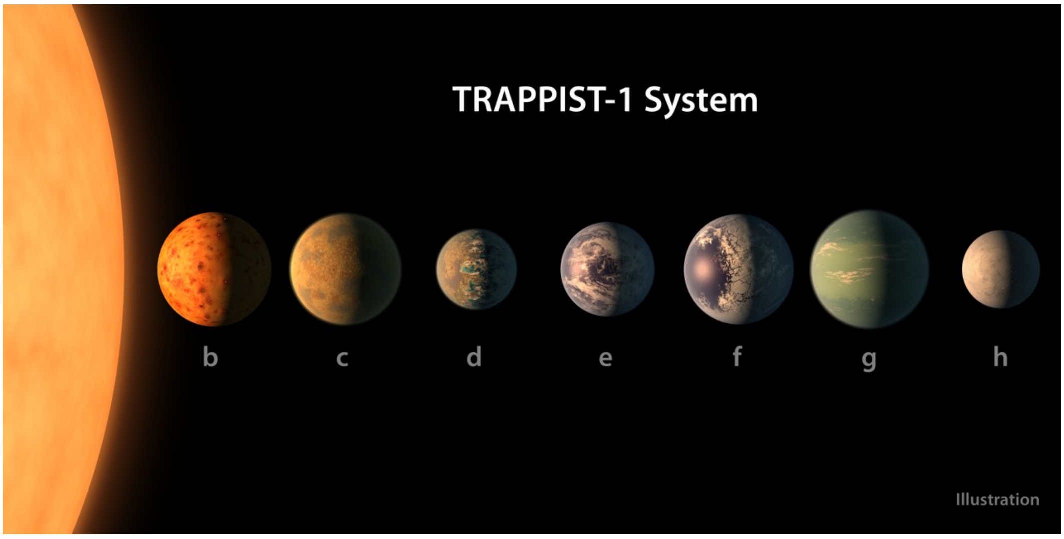

Figure 4: A size comparison of the planets of the TRAPPIST-1 system, lined up in order of increasing distance from their host star. The planetary surfaces are portrayed with an artist's impression of their potential surface features, including water, ice, and atmospheres. Credit: NASA/R. Hurt/T. Pyle

Data about the system is below:

	Mass / M_E	Radius / R_E	Orbital Period / Earth days
TRAPPIST-1b	1.02	1.127	1.511
TRAPPIST-1c	1.16	1.100	2.422
TRAPPIST-1d	0.30	0.788	4.050
TRAPPIST-1e	0.77	0.915	6.099
TRAPPIST-1f	0.93	1.052	9.206
TRAPPIST-1g	1.15	1.154	12.354
TRAPPIST-1h	0.33	0.777	18.768

The star has a mass of $0.089\,M_\odot$, radius $0.121\,R_\odot$, luminosity $5.22 \times 10^{-4}\,L_\odot$ and effective surface temperature of 2511 K. Assume the planets orbit in circular, coplanar orbits.

For black-body radiation (which is a good approximation for a stellar spectrum), the modal wavelength in the spectrum (in terms of intensity), $\lambda_{\rm max}$, is related to the effective temperature, T, through Wien's displacement law,

$$\lambda_{\rm max} T = 2.90 \times 10^{-3} \text{ m K} .$$

The smallest angular width, $\theta_{\rm min}$, in radians that a telescope can resolve is limited by diffraction (ignoring the effects of the atmosphere). For a telescope of aperture diameter D, operating at wavelength λ, it can be shown that

$$\theta_{\rm min} = \frac{1.22\lambda}{D} .$$

(a) Verify that the central star, as viewed from planet d, is of comparable brightness (i.e. intensity, measured in W m^{-2}) to the Sun as viewed from Earth, and hence determine the apparent magnitude of the star. [The apparent magnitude of the Sun as viewed from Earth is -26.74]

[5]

(b) Assuming life developed on both planets d and f, calculate the size of telescope an alien on planet f would need to be able to resolve a city 20 km across on planet d when they are at their closest in their orbits. Is such a telescope feasible? [Assume their eyes have evolved to operate around the modal wavelength of the star.]

[5]

[Hint: You may find the small angle approximation useful: $\tan x \approx x$ when $|x| \ll 1$ and in radians. A radian is a measure of angle, and there are 2π radians in a circle (i.e. 2π radians $= 360°$).]

END OF PAPER

Questions proposed by:
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