SPONSORED BY
RESEARCH

BPhO Round 1

Section 2

8^{th} November 2024

This question paper and any notes must not be taken out of the exam room

Instructions

Time: 1 hour 20 minutes.

Section 2 - **Only answer two questions**, which are worth 25 marks each.

Students are recommended to spend about 40 minutes on each question.
Each question contains independent parts so that later parts should be attempted even if earlier parts are incomplete.

Working: Working, calculations, explanations of the physics and **diagrams**, properly laid out, must be shown for full credit. The final answer alone is not sufficient. Writing must be brief but clear. If derivations are required, they must be mathematically supported, with any approximations stated and justified. Marks are given for intermediate steps if they can be seen: underline or circle them so that the marker can find them.

Instructions: You are allowed any standard exam board data/formula sheet.

Calculators: Any standard calculator may be used, but calculators must not have symbolic algebra capability. If they are programmable, then they must be cleared or used in "exam mode". Code may not be written for any of the BPhO competitions.

Solutions: **1.** Answers and calculations are to be written on loose paper **ON ONE SIDE ONLY** (pages will be scanned). **2.** Students should write their **name** and their **school/college** clearly on every answer sheet. **3.** Number each question clearly. **4. Number your pages** at the top. **5.** Write "END" at the end of your script. **6.** Fill in the Front Cover Sheet your teacher will give you - **just one cover sheet for the two sections**.

Setting the paper: There are two options for sitting BPhO Round 1:

a. *Section 1* and *Section 2* may be sat in one session of 2 hours 40 minutes *Section 1* should be collected in after 1 hour 20 minutes and then *Section 2* given out.

b. *Section 1* and *Section 2* may be sat in two sessions on separate occasions, with 1 hour 20 minutes . If the paper is taken in two sessions on separate occasions, *Section 1* must be collected in after the first session and *Section 2* handed out at the beginning of the second session.

Important Constants

Constant	Symbol	Value
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Mass of electron	m_{e}	$9.110 \times 10^{-31}\,\mathrm{kg}$
Mass of proton	m_{p}	$1.673 \times 10^{-27}\,\mathrm{kg}$
Mass of neutron	m_{n}	$1.675 \times 10^{-27}\,\mathrm{kg}$
atomic mass unit	u	$1.661 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV\,c^{-2}}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^{3}\,kg^{-1}\,s^{-2}}$
Earth's gravitational field strength	g	$9.81\,\mathrm{N\,kg^{-1}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Avogadro constant	N_{A}	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Gas constant	R	$8.3145\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Mass of Sun	M_{S}	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Earth	R_{E}	$6.37 \times 10^{6}\,\mathrm{m}$
Specific heat capacity of water	c_{w}	$4180\,\mathrm{J\,kg^{-1}\,{}^{\circ}C^{-1}}$

$$T_{(\mathrm{K})} = T_{(^{\circ}\mathrm{C})} + 273$$

$$\text{Volume of a sphere} = \tfrac{4}{3}\pi r^3$$

$$\begin{aligned}
e^x &\approx 1 + x + \ldots & x &\ll 1 \\
(1+x)^n &\approx 1 + nx & x &\ll 1 \\
\frac{1}{(1+x)^n} &\approx 1 - nx & x &\ll 1 \\
\tan\theta &\approx \sin\theta \approx \theta & \text{for } \theta &\ll 1 \\
\cos\theta &\approx 1 - \frac{\theta^2}{2} & \text{for } \theta &\ll 1
\end{aligned}$$

Section 2 — Attempt two questions only

You may be able to do later parts of a question even if you cannot do the earlier parts.
Approximate marks are shown for each section.

Question 2

This question is about solar energy, energy losses and capacitors.

a) A simple experiment to estimate solar power output is conducted by a student on a clear sunny day in Spain.
The setup is shown in **Fig. 1**, and consists of a thin aluminium walled matte black can filled with $300\,\mathrm{g}$ of water. A thermometer, fitted through a small hole in the black painted lid, measures the temperature of the water. The can, lid, water and thermometer are allowed to come to ambient temperature in the shade, before being placed on an insulating surface in full sunlight.

Figure 1: A water filled can with a lid and thermometer, sitting on an insulated base in the sunshine.

After 20 minutes, the temperature of the water in the can has risen from $27.0\,°\mathrm{C}$ to $30.0\,°\mathrm{C}$. The angle of the sun to the horizontal is $50°$. The can has a diameter of $6.0\,\mathrm{cm}$ and a height of $10.0\,\mathrm{cm}$.
Neglecting the heat capacity of the can, lid and thermometer, calculate a value for the intensity ($\mathrm{W\,m^{-2}}$) of the Sun's radiation.
Ignore energy losses from the can. Assume that the can and lid absorb all of the incident radiation.

[7]

b) (This part is about the capacitor circuit only, which is a model). A student believes that it might be possible to model the behaviour of the energy transfer above, by considering the can of water to be a capacitor being charged up with charge Q, and the energy losses to be a leak of current across the capacitor as shown in **Fig. 2**. We can determine the time constant for the circuit shown in terms of r, R and C.

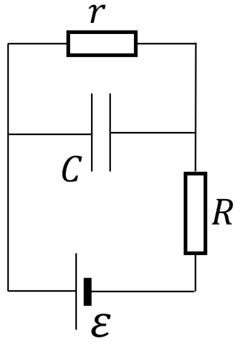

Figure 2: Circuit with a "leaky" capacitor.

(i) Write down the potential around the circuit, in terms of ε, the current I through R, and the potential across the capacitor from C and its charge, Q.

(ii) Now write down the current through R (and the cell), I, in terms of the current through r (using the potential on C) and the current flowing onto C, both in terms of Q.

(iii) Hence write an equation for ε in term of $\frac{\mathrm{d}Q}{\mathrm{d}t}$ and Q.

(iv) What is the expression for the time constant τ of this capacitor circuit? (You may read this off the differential equation without doing any integration).

(v) By observation of the circuit diagram rather than an equation, what would be the steady state (equilibrium) charge, Q_{ss}, on the capacitor after a long time, in terms of R, r, ε and C?

(vi) Sketch a graph of the charge on the capacitor against time, marking on ε and Q_{ss}.

[6]

c) Since the power reaching the can of water is constant, in our model the cell in the **Fig. 2** is replaced with a constant current supply (the device adjusts the voltage output to maintain a specified current). The circuit becomes that of **Fig. 3**, in which the source current is I_0.

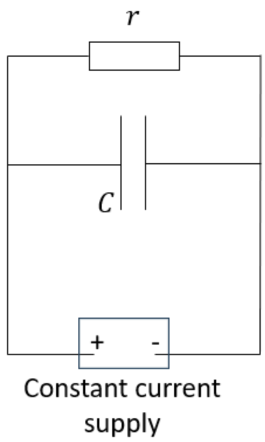

Figure 3: Circuit with a "leaky" capacitor and constant current supply.

(i) After a long time, the circuit of **Fig. 2** will reach a steady state. By observation of the circuit diagram what will be the maximum charge on the capacitor, and its energy stored, in terms of I_0, r and C?

The rate of flow of charge to the capacitor is analogous to the rate of increase in internal energy of the water in the can. The current through the resistor r is analogous to the power lost to the surroundings, given by $P_{\mathrm{leak}} = kT$, where T is the temperature difference between the water in the can and the surroundings and k is a constant.

(ii) Write down an equation for the power absorbed by the can, P_{can} in terms of the rate of temperature increase of the water, the specific heat capacity of the water c, the mass of water m, the temperature excess T and k.

(iii) Using your result from part (ii), what is the simple expression for the steady state result for P_{can}?

(iv) Calculate the value of k given that over 40 minutes the temperature of the can falls from $34.2\,^{\circ}\text{C}$ to $32.8\,^{\circ}\text{C}$ in an ambient temperature of $31.0\,^{\circ}\text{C}$.

(v) Produce a new estimate for the power output of the Sun, this time accounting for energy losses from the can.

[7]

d) The solar constant is the intensity of the Sun's radiation at the top of the atmosphere, normal to the radial line between the Earth and the Sun and has a value of $1360\,\text{W}\,\text{m}^{-2}$. The Sun lies at a distance of $1.5 \times 10^{11}\,\text{m}$ from the Earth and has a radius of $696\,000\,\text{km}$.

(i) What is the power produced on average per cubic metre within the Sun?

(ii) Using the equation of mass-energy equivalence, $E = mc^2$, estimate how much mass the Sun loses each second through radiation.

[5]

[25 marks]

Question 3

This question is about bubbles, forces and rotation.

a) A spherical soap bubble is filled with warm air but, due to the weight of the liquid film, the bubble sinks slowly at speed v in the surrounding cooler air. As the soap film evaporates, the bubble becomes lighter and eventually starts to rise. We wish to determine the length of time it takes between it falling at speed v in still air and then, a short time later, the bubble rising in still air at speed v.

Figure 4: A bubble floating in air.

The heavier (sinking) and lighter (rising) masses of the bubble are m_{h} and m_{ℓ} respectively.
The bubble has a radius $r = 8.0\,\mathrm{cm}$, the density of the soap solution is $998\,\mathrm{kg\,m^{-3}}$, and the soap film evaporates from the outer surface at a rate of $2.4 \times 10^{-5}\,\mathrm{kg\,m^{-2}\,s^{-1}}$.

(i) Using the fact that the magnitude of the drag force, F_{d}, is the same when the bubble is falling as when it is rising, obtain two equations, for the buoyancy, B, and for F_{d}, acting on the bubble in terms of m_{h}, m_{ℓ} and g.

(ii) Calculate the magnitude of the buoyancy force, when the air inside is at $25\,^{\circ}\mathrm{C}$ and the cooler air in the room is at $17\,^{\circ}\mathrm{C}$. Take atmospheric pressure as $1.01 \times 10^{5}\,\mathrm{Pa}$ and the molar mass of the air inside and outside to be to be $28\,\mathrm{g\,mol^{-1}}$.
Ignore the extra pressure due to the soap film, and the water vapour inside the bubble.

(iii) The slow speed of motion means that $F_{\mathrm{d}} = kv$ with $k = 1.2 \times 10^{-3}\,\mathrm{N\,s\,m^{-1}}$. Calculate how long it would take for sufficient mass of the soap film to have evaporated from the outside surface of the bubble so that the velocity changes from $6.0\,\mathrm{cm\,s^{-1}}$ downwards to $6.0\,\mathrm{cm\,s^{-1}}$ upwards.

(iv) Calculate the initial thickness of the bubble wall.

[10]

b) (i) Four identical masses, m, are attached to each other by four light strings of equal length ℓ, as in **Fig. 5**.

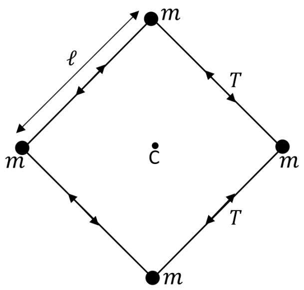

Figure 5: Deriving the tension in a rotating set of masses.

The system rotates in the plane of the masses about the centre C, with each mass moving at speed v in a circle.
Obtain expressions for the radial T_{r} and tangential $T_{\mathrm{tangential}}$ components of the tension T in the string, in terms of m, v and ℓ.

When the masses are not discrete but continuous, a modified approach is used.

(ii) A thin circular hoop of mass m and radius r is rotated in a horizontal plane about its centre with speed v as in **Fig. 6**. The hoop has a mass per unit length (linear density) of λ.

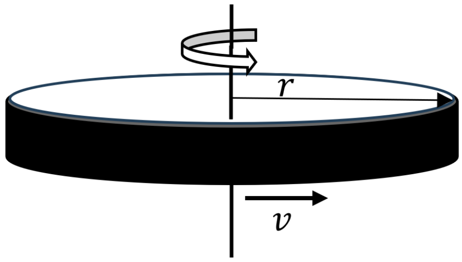

Figure 6: A thin hoop rotating at speed v. The tension $T_{\text{tangential}}$ in the hoop prevents it breaking apart as it rotates.

The tension $T_{\text{tangential}}$ in the hoop holding it together is given by the expression

$$T_{\text{tangential}} = k\lambda^{\alpha}v^{\beta}r^{\gamma}$$

where k is a dimensionless quantity. By considering the dimensions (or units) of the terms in this expression, and assuming $k = 1$, obtain an equation for $T_{\text{tangential}}$.

(iii) A rubber band of mass $25\,\text{g}$ which behaves linearly, has an unstretched length of $15.0\,\text{cm}$ and a spring constant of $2000\,\text{N}\,\text{m}^{-1}$. It is stretched round a horizontal flywheel (a disc) of circumference $18.5\,\text{cm}$. At what speed of rotation of the disc will the band fall off?

[10]

c) A thin, elastic ring of mass m and radius r of **Fig. 7** oscillates at its lowest natural frequency in a radial direction, expanding and contracting. The circumferential tension in the ring is proportional to the extension of the circumference i.e. $T = k\delta x$ where δx is the extension of the ring circumference and k is a constant. Determine an expression for frequency, f, of radial oscillations in terms of λ (the mass per unit length), r and k.

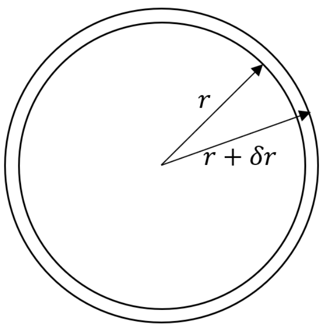

Figure 7: Radial oscillations of a ring.

[5]

[25 marks]

Question 4

This question is about the physics of landing a craft on Titan.

a) A Radioisotope Thermoelectric Generator (RTG) uses the heat produced during radioactive decay as a source of energy, (as in NASA's Dragonfly expedition to Saturn's large moon Titan).
The core of a typical unit consists of $4.8\,\mathrm{kg}$ of plutonium-238 dioxide. The energy is produced by the $5.5\,\mathrm{MeV}$ alpha decay of Pu-238, with a half-life of $87.7\,\mathrm{years}$. Calculate the initial thermal power output.

[3]

b) The ambient temperature on Titan is $-179\,^{\circ}\mathrm{C}$. The inside of the landing craft is maintained at $0\,^{\circ}\mathrm{C}$, insulated with $5.0\,\mathrm{cm}$ thick Rohacell foam that has a thermal conductivity of $0.035\,\mathrm{W\,m^{-1}\,K^{-1}}$. If the internal heat source is producing $2000\,\mathrm{W}$ of power, estimate the internal area of the lander.

Hint: the rate of heat flow through a conductor, $\frac{\mathrm{d}Q}{\mathrm{d}t}$, is proportional to the cross sectional area it flows through, A_c, the temperature difference, ΔT, and inversely proportional the thickness of conductor, Δx. The constant of proportionality is the thermal conductivity.

[2]

c) The landing craft is equipped with eight rotors for the purpose of flying as a drone.

(i) The force acting on the air by all eight rotor blades, F_8, is used to lift and drive the craft. Give an expression for the hovering lift force, F_lift, in terms of the speed of the air downwards, v, the density of the atmosphere on Titan, ρ_T, and the area swept out by the eight rotors, A_8.

(ii) Hence obtain an expression for the power, P, needed to hover, in terms of the mass of the craft m_D, the gravitational field strength on Titan g_T, A and ρ_T.

(iii) Using the data below, calculate the ratio of the power needed to hover on Titan to the power needed to hover on Earth, $P_\mathrm{T}/P_\mathrm{E}$:

(iv) Calculate the hover power required by the craft on Titan.

ρ_T	$5.35\,\mathrm{kg\,m^{-3}}$
ρ_E	$1.22\,\mathrm{kg\,m^{-3}}$
g_T	$1.35\,\mathrm{N\,kg^{-1}}$
g_E	$9.81\,\mathrm{N\,kg^{-1}}$
m_D	$420\,\mathrm{kg}$
length of each rotor blade	$0.68\,\mathrm{m}$

(v) The efficiency of the conversion of heat to electrical energy is too low to allow the RTG to power flight. Estimate how long the onboard battery would need to be charged using the RTG at an efficiency of 6%, in order to complete a flight of $8\,\mathrm{km}$

at an altitude of $500\,\text{m}$ at $10\,\text{m}\,\text{s}^{-1}$, including ascending and descending. Assume the density of the atmosphere does not change over this altitude, there are no losses due to drag, the craft can also rise and fall vertically at a constant $10\,\text{m}\,\text{s}^{-1}$. In ascending it consumes power at its hovering rate plus the GPE gained, whilst in descending it consumes power only at its hovering rate.

[10]

d) The rotor driving force, F_8, is a function of ρ_T, A_8, and the rotation speed of the tips of the rotors, v_tip, and is given by $F_\text{drive} = C_\text{L}\rho_\text{T}A_8v_\text{tip}^2$, where $C_\text{L} \approx 1$.
The drag force, D, on the landing craft can be approximated in a similar way, with $D = C_\text{D}\rho_\text{T}A_8v_\text{air}^2$,where v_air the airspeed of the craft and $C_\text{D} \approx 1$.

(i) When the craft is at constant speed in horizontal flight the driving force, F_8, of the rotors is required to counter the drag force by providing thrust T, and countering the weight, W, of the craft by providing lift, L. Show all five of these listed forces, T, D, W, L, F_8, on a diagram and write down an expression for the magnitude of F_8 in terms of D and W.

(ii) Substituting the expressions for each of these forces, obtain an expression for the airspeed, v_air, in terms of $\rho_\text{T}, A, m_\text{D}, g_\text{T}$ and v_tip.

(iii) From your expression calculate the velocity for horizontal flight given a rotor speed of $500\,\text{rpm}$.

[6]

e) The space capsule containing the Dragonfly landing craft enters the atmosphere of Titan and for much of its travel through the outer atmosphere of the moon the velocity is approximately constant at $7300\,\text{m}\,\text{s}^{-1}$, as seen in the graph of **Fig. 8**. The resistance to motion provided by the tenuous atmosphere is determined by the turbulent flow of the atmosphere around the capsule.

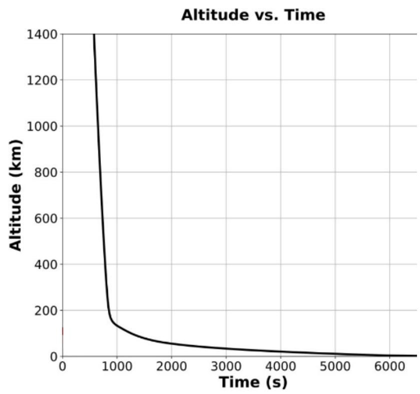

Figure 8: Graph of Dragonfly entering the Martin atmosphere and landing.

(i) The resistive force acting on the approximately spherical capsule, assuming turbulent flow, will be given by $F = k\rho^\alpha v^\beta r^\gamma$,
where k is a dimensionless constant, ρ is the density of the atmosphere of Titan, v is the speed of the capsule, and r is the radius of the capsule.
Using dimensions or otherwise, determine the coefficients α, β, γ and write the equation for F.

(ii) Taking $k = 1$ and the diameter of the capsule to be $3.7\,\text{m}$, estimate the density of the atmosphere at an altitude of $600\,\text{km}$.

Mass of Titan $= 1.345 \times 10^{23}\,\text{kg}$
Mass of capsule $= 400\,\text{kg}$
Radius of Titan $= 2575\,\text{km}$

[4]

[25 marks]

Question 5

This question is about gravity and collisions.

a) Consider a planet of mass m_p, orbiting a star of mass M_s in a circular orbit of radius r which takes a time T to complete one orbit.

(i) By balancing centripetal and gravitational forces, show that the orbital speed $v \propto r^{-1/2}$.

(ii) Hence, show that the total energy of the planet (KE + GPE), $E_\text{total} \propto r^{-1}$.

(iii) By considering the speed of a planet in its orbit around the planet in terms of r and T, show that the relationship between orbital period and orbital radius is $T^2 \propto r^3$. (This is known as Kepler's Third Law.)

(iv) Given the Earth has an orbital radius $r_\text{E} = 1.50 \times 10^{11}\,\text{m}$ and orbital period $T_\text{E} = 365$ days, calculate the mass of the Sun in kg.

Earth's orbital properties can be given in different units, namely $T_\text{E} = 1$ year, and $r_\text{E} = 1\,\text{au}$. (An "au" stands for astronomical unit).

(v) Using your result from part (iii), if you use the Earth values of $r_\text{E} = 1\,\text{au}$ and $T_\text{E} = 1\,\text{year}$ orbiting a star of mass $M_\text{s} = 1\,M_\text{Sun}$, what is the value (and units) of G in just these given quantities and units?

[5]

For the following calculations, you may choose to work in standard SI units like metres, kilograms and seconds (and so use your value of M_Sun from part (iv)), or in au, solar masses and years, as that can often greatly simplify the formulae (and so use your value of G from part (v)).

b) Disaster strikes and the Sun disappears! This causes all the planets to be perturbed from their orbits. In this question we will just look at Venus and Earth. Assume that the loss of the Sun's gravity was felt instantaneously by both planets, and that both are travelling in coplanar circular orbits.

Venus has a mass of $0.815\,M_\text{Earth}$, where $M_\text{Earth} = 5.97 \times 10^{24}$ kg, and an orbital radius of $0.723\,\text{au}$.

Venus and Earth happened to be at such a point in their orbit when the Sun disappeared that they are on a collision course.

(i) Calculate the orbital speeds of Earth and Venus, v_E and v_V.

(ii) Determine the time, t, until they crash. Give your answer in days.

(iii) Show that the distance d from the Sun's original location when they crash is ~ 1.5 au.

(iv) Show that the angle between their velocity vectors as they approach is $\sim 13°$.

(v) Find the Earth-Sun-Venus angle at the moment the Sun disappeared in order for this crash to occur.

[8]

c) The collision between them is inelastic, causing them to stick together and make a new, heavier planet: "Venearth".

(i) Show that the speed of Venearth, v, just after the collision is ~ 6.7 au year^{-1}.

(ii) Determine the angle between Venearth's velocity vector and Earth's initial velocity vector before the collision.

(iii) Calculate the percentage of kinetic energy lost in the collision.

[7]

d) At the moment of the collision, a new star magically appears in the location that the Sun used to be. Again, assume its gravitational effects are felt instantaneously throughout the Solar System.

(i) What is the minimum mass of this new star necessary for Venearth to still be bound within the Solar System? Give your answer in units of M_{Sun}.

The new planet Venearth will follow an elliptical orbit whose orbital period will be equal to a circular orbit with the same total energy as Venearth.

(ii) If in fact, the new star has a mass equal to M_{Sun}, what will be the orbital period of Venearth? Give your answer in years.

[5]

[25 marks]

END OF SECTION 2