Astro Round 1 2025 MS (Summary version)

General note:

- Check final answer first – if correct with no major Physics errors in their working that jump out at you, award full marks. No penalty for slight rounding errors
- Where possible, try and apply error carried forward. **Look out for standalone marks**
- Correct answer but not in the units specified by the question = lose 0.5 marks

Section 1

A)

- distance from Sun = 0.575 (pc)
- angular diameter = 16.1 mas (must be in mas)

[2]

B)

- helpful diagram of the geometry of the situation
- Sun must move through an angle of 1.176 rad
- time is 4h 30m (must be in hours and mins)

[3]

C)

- realise that the correct distance to use in vis viva is 3.83e6 miles + R_sun
- semi-major axis (5.81e10 m)
- period of 88.4 days (must be in days)

[3]

D)

- calculate required intensity is $\frac{mc\Delta T}{t} = 320 \text{ kW m}^{-2}$
- recognise that the area (of the lens) will cancel
- link to standard lens geometry
- distance from lens is 18.7 cm

[4]

E)

- calculate Sun's angular diameter (0.53°)
- appropriate triangle
- link angle in triangle to the one that represents the latitude
- angle of great circle covered = 1.32°
- latitude = 66.2° (N)

[5]

F)

- convert 3.13 eV into a wavelength of 397 nm
- use Hubble's law to get recessional velocity of 25550 km/s
- redshift of 0.085
- new wavelength of 431 nm
- new energy of 2.88 eV (must be in eV)

[5]

G)

- distance to star = 7.69 pc
- absolute magnitude = 1.73
- convert to luminosity = 16 L_Sun (= 6.13e27 W)
- Wien's displacement law to get temperature = 8580 K
- radius = 1.81 R_Sun (must be in R_Sun)

[5]

H)

i)

- y-coordinate of Y1: $y = b\sqrt{1 - e^2}$
- differentiate expression to get $y' = \frac{b^2(ae-x)}{a^2 y}$
- evaluate y' at Y1 to get $\frac{b}{a}\frac{e}{\sqrt{1-e^2}}$
- use $e = \sqrt{1 - \frac{b^2}{a^2}}$ to simplify it to $\boxed{y' = e}$ at Y1
  (note this simplification may happen earlier e.g. the y-coordinate at Y1 is $y = \frac{b^2}{a}$)

[4]

ii)

- distance = 2a

[1]

I)

- convert 52.7" into a physical distance across the Sun of 3.82e7 m
- convert 8' into a solar latitude of 30° (N)
- method to correct for solar latitude
- corrected vertical distance of 4.41e7 m
- use πab to get area as 1.32e15 m^2
  (watch out for them forgetting the factor of 2 in going from major and minor axis length to semi-major and semi-minor axis)
- Earth surface area = 5.1e14 m^2
- Sunspot area = 2.60 Earth surface area (must be in this unit – no 0.5-mark compensation)

[7]

J)

i)

- Express $\hbar$ **or** G in correct base units ([$\hbar$] = kg m^2 s^-1 and [G] = m^3 kg^-1 s^-2)
- $\alpha = 0.5$
- $\beta = 0.5$
- $\gamma = -1.5$

  (this can all be encapsulated by the expression $L_p = \sqrt{\frac{\hbar G}{c^3}}$, although if this is just quoted without working of dimensional analysis then award no marks here)

[4]

ii)

- Value for Planck length = 1.61e-35 m
- Use of Schwarzschild radius formula to get radius = 1.27e10 m
- Value of entropy = 2.69e67 J/K (incorrect [or absent] units on final answer = lose 0.5 marks)

[3]

K.

i)

- relative angular velocity $\omega_{rel} = \omega_A - \omega_B \therefore \frac{2\pi}{T_{rel}} = \frac{2\pi}{T_A} - \frac{2\pi}{T_B} \therefore \frac{1}{T_{rel}} = \frac{1}{T_A} - \frac{1}{T_B}$

  (allow this mark for any method that will work to find the synodic period)
- time period between transits = 300 days

[2]

ii)

- helpful diagram
- clear algebraic method to find the duration
- transit time = 8.11 hours (must be in hours)

[3]

iii)

- the transit for the inclined orbit is SHORTER [this mark can be awarded even if no working]
- helpful diagram
- relative angular velocity = 0.03215 rad/day
- transit time = 5.29 hours
- so the difference in transit time is 2.83 hours (must be in hours)

[5]

Section 2 **(Bullet point = mark)**

Q2 – Spaceship Strategies

a) One mark for each correct expression

i)

- $v_p = \sqrt{GM\left(\frac{2}{a(1-e)} - \frac{1}{a}\right)}$

ii)

- $v_p = \sqrt{GM\left(\frac{2}{a(1+e)} - \frac{1}{a}\right)}$

iii)

- $\frac{v_p}{v_a} = \frac{1+e}{1-e}$

b)

- $\Delta v = \sqrt{\frac{2GM}{r}} - \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$ (use of escape velocity and vis-viva)
- $\Delta v = \sqrt{\frac{2GM}{a_1(1-e)}} - \sqrt{GM\left(\frac{2}{a_1(1-e)} - \frac{1}{a_1}\right)}$
- $k_A = \sqrt{\frac{2}{1-e}} - \sqrt{\frac{2}{1-e} - 1}$

c)

- $k_B = \sqrt{\frac{2}{1+e}} - \sqrt{\frac{2}{1+e} - 1}$

d)

i)

- $a_1(1+e) = a_2(1-e) \therefore a_2 = a_1\frac{1+e}{1-e}$
- $\Delta v_1 = \sqrt{GM\left(\frac{2}{a_1(1+e)} - \frac{1}{a_2}\right)} - \sqrt{GM\left(\frac{2}{a_1(1+e)} - \frac{1}{a_1}\right)}$
- $k_1 = 1 - \sqrt{\frac{1-e}{1+e}}$
- any suitable test to show that $k_1 \leq k_B$

ii)

- Identify that it's a geometric sum with first term $\sqrt{\frac{GM}{a_1}}\left(1 - \sqrt{\frac{1-e}{1+e}}\right)$ and common ratio $\sqrt{\frac{1-e}{1+e}}$
- Sum = $\sqrt{\frac{GM}{a_1}}$ so $\boxed{k_C = 1}$
  (so this strategy is less fuel efficient than Spaceship B for all e, expect e = 1 where they're equal – expect to see this statement, although no penalty if it's missing)

e)

i)

- Time between n and $n+1$ burst $= \frac{1}{2}T_{n+1} = \frac{1}{2}a_{n+1}^{3/2} = \frac{1}{2}\left(a_1\left(\frac{1+e}{1-e}\right)^n\right)^{3/2}$
- Spot that for every burst to be at opposition, need the time between bursts to be an odd integer times by 0.5 Lottes (since only half a transfer ellipse is between each burst)
- Need $\left(a_1\left(\frac{1+e}{1-e}\right)^n\right)^{3/2}$ to be an odd integer so need a_1 and $\left(\frac{1+e}{1-e}\right)$ to be odd square integers (i.e. 1, 9, 25, 49... etc.)
- Given the conditions on a_1 and e, they must be $\boxed{a_1 = 1 \text{ Char}}$ and $\left(\frac{1+e}{1-e}\right) = 9 \therefore \boxed{e = 0.8}$ (0.5 marks for each)

ii)

- $T = \frac{1}{2}\left(a_1\left(\frac{1+e}{1-e}\right)^1\right)^{3/2} = \frac{1}{2}(1 \times 9)^{3/2} = \boxed{13.5 \text{ Lottes}}$

iii)

- First and second burst distances are $1(1+0.8) - 1 = 0.8 \text{ Char}$ and $9(1+0.8) - 1 = 15.2 \text{ Char}$ (0.5 marks for each – lookout for subtracting the planet's orbital radius from apoapsis distance)
- $\Delta m = 5\log\left(\frac{15.2}{0.8}\right) = \boxed{6.4 \text{ mag fainter}}$

Q3 – Black Dwarfs

a)

- $g = 10^{5.771} = 5.90 \times 10^5 \text{ m s}^{-2}$
- $R_{WD} = \sqrt{\frac{GM_{WD}}{g}} = 1.05 \times 10^7 \text{ m} = \boxed{1.65\ R_\oplus}$ (must be in $R_\oplus$)

b)

- Absolute magnitude = 13.4
- Luminosity = 3.50e-4 L_Sun = 1.34e23 W
- Use Stephan-Boltzmann Law to get T = 6427 K
- This will appear white / white-blue [Do not accept green or blue]
  (If they have not got this mark, but worked out max wavelength as 451 nm, give 0.5 marks)

c)

- Balance P_in and P_out (at equilibrium temperature)
- Use this to derive $T = \sqrt[4]{\frac{L_{WD}(1-\alpha)}{16\pi\sigma a^2}}$ or alternatively $T = T_{WD}\sqrt{\frac{R_{WD}}{2a}}\sqrt{1-\alpha}$
  [If either of these formulae are quoted without derivation, get both first and second mark]
- Equilibrium temperature = 24.2 K

d)

- Correct rearrangement and manipulation of logs
- White dwarf age = 1.88e9 year = 1.88 Gyr (must be in Gyr)

e) Needs to be 2 marks

- Using Wien's displacement law with peak wavelength to get T = 797 K
- Minimum time to get a black dwarf = 1.14e12 years = 102 t_0 (must be in t_0)

f) Needs to be 7 marks

- Speed of planet in orbit, $v = \sqrt{\frac{GM}{a}} = 14.47 \text{ km s}^{-1}$
- Gravitational radiation timescale = 5.16e29 s (= 1.64e13 Gyr)
- Convert n from 0.1 pc^{-3} to 3.39e-51 m^{-3}
- Ejection timescale = 9.74e21 s (= 3.09e5 Gyr)
- Therefore, ejection is the most likely outcome for this planet
- Rearrangement to get $a = \sqrt[6]{\frac{(GM_{WD})^3}{2\pi^2 c^5 n\, v_{rel}}}$
- Timescales equal at a distance of 1.60e10 m (= 0.107 au)

Q4 – International Liquid Mirror Telescope

a) From the Rayleigh Criterion:

[3 marks]

$$\theta_c = 1.22\frac{\lambda}{d} = 1.22 \times \frac{7.0\times10^{-7}}{4.0} = 2.14\times10^{-7}\text{ rad} = 0.044''$$ [1]

For the actual resolution, we need to find the angle subtended by each pixel at the primary mirror. Using small angle approximation

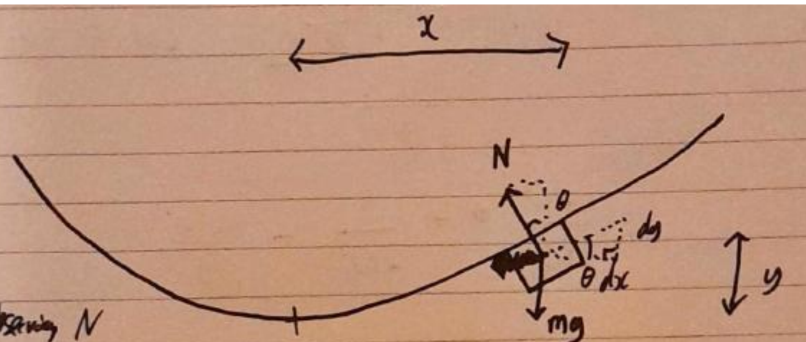
$$\theta \approx \frac{\text{pixel width}}{\text{focal length}} = \frac{13\times10^{-6}}{8.0} = 1.63\times10^{-6}\text{ rad} = 0.34''$$ [1]

This is 7.6× the Rayleigh Criterion
(So the resolution is limited by pixel size, not diffraction)

[1] (only ratio calculation needed for mark)

b)

[6 marks]



[1] for observing N perpendicular to surface

[1] for force diagram with only N and mg

[1] for correct centripetal force

[1] for equating centripetal force with mg tanθ

[1] for $\tan\theta = \frac{dy}{dx}$

[1] for correct integration

For a parcel of mercury on the surface, the only forces acting are a fluid pressure perpendicular to the surface, and the parcel's weight.

For centripetal acceleration: $mg\tan\theta = m\omega^2 x$

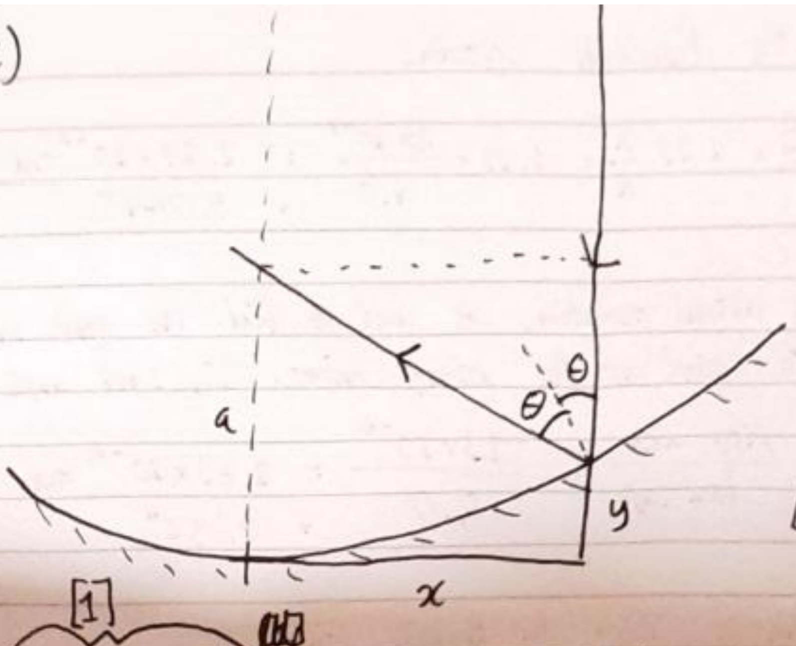
Where θ is the angle the surface makes with the horizontal, so $\tan\theta = \frac{dy}{dx}$

$$\therefore mg\frac{dy}{dx} = m\omega^2 x$$

$$\frac{dy}{dx} = \frac{\omega^2}{g}x \Rightarrow y = \frac{\omega^2 x^2}{2g}$$ (as $y=0$ when $x=0$)

(c)

[6 marks]



[1] for useful diagram

[1]

$$a = y + \frac{x}{\tan 2\theta} = y + \frac{x(1-\tan^2\theta)}{2\tan\theta}$$

[1] for subs y
[1] for subs tan θ

$$= y + \frac{x\left(1-\left(\frac{\omega^2 x}{g}\right)^2\right)}{\frac{2\omega^2 x}{g}}$$

$$= \frac{\omega^2 x^2}{2g} + \frac{g}{2\omega^2} - \frac{\omega^2 x^2}{2g}$$

$$= \frac{g}{2\omega^2} \quad [1]$$

Since this is independent of x and y, all vertical light rays focus to the same point, so a is the focal length

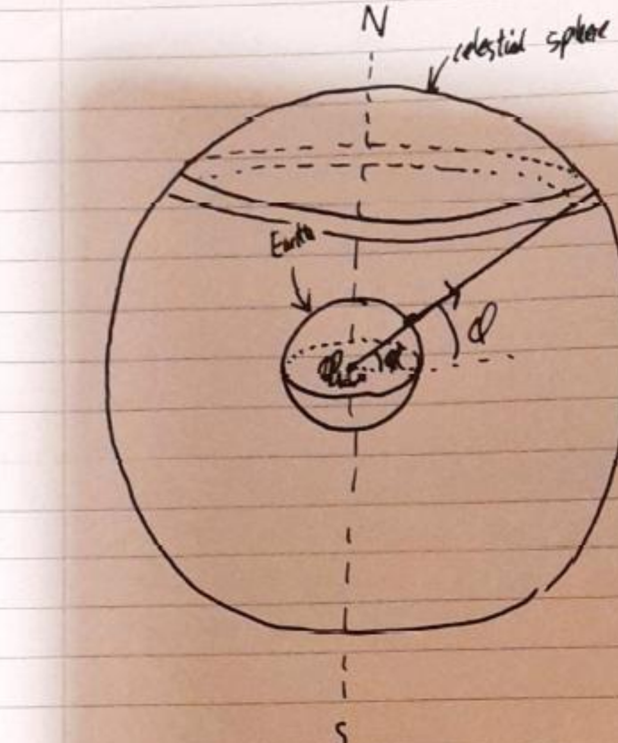
$$\omega = \sqrt{\frac{g}{2a}} = \sqrt{\frac{9.81}{2\times 8.0}} = 0.783\ s^{-1}$$

$$T = \frac{2\pi}{\omega} = 8.0\ s \quad [1]$$

[5 marks]

(d) The key observation here is that because the ILMT relies on gravity to maintain the shape of the mirror, it can only ever point directly up. So only the sky in a narrow strip can be imaged, because the zenith only moves around a circle corresponding to the telescope's lattitude, ϕ

[1] for considering of a strip



This strip has length $2\pi R\cos\phi$, and width [1]

$R\delta$, where δ is the FOV of the telescope.

We could calculate δ directly using the size of the plate, but instead note it is 4000× the angle θ calculated in (a) along a side (as we have 4000 pixels, each covering an angle θ) [1]

$$\therefore \% \text{ of sky} = \frac{(2\pi R\cos\phi)\times(4000\,R\theta)}{4\pi R^2}\times 100\%$$

$$= \frac{4000\,\theta\cos\phi}{2}\times 100\%$$

$$= 0.28\%$$ [1]

Since we can only observe each point when it is far away from the sun (we can't take pictures of stars in the day!) the observations must take place over ~ 1 year at minimum, to allow the sun to move round [1]

(Allow any well reasoned answer between 0.5 – 1 years)

Q5 – Big Bang nucleosynthesis

a)

- Use ratio of densities to get z = 1e8
- Hence get T_0 = 6 K

b)

- Temperature at decoupling 2974 K
- Energy of photons 0.26 eV (must be in eV)
- Wavelength of photons 4.85 μm
- They are INFRARED

c)

- Use $1 + z \propto t^{-3/2}$ in a ratio form OR calculate the constant of proportionality
- Start of decoupling at $z = 1090 + \frac{195}{2}$ corresponding to 3.26e5 years OR end of decoupling at $z = 1090 - \frac{195}{2}$ corresponding to 4.27e5 years
- Duration of decoupling = 1.01e5 years

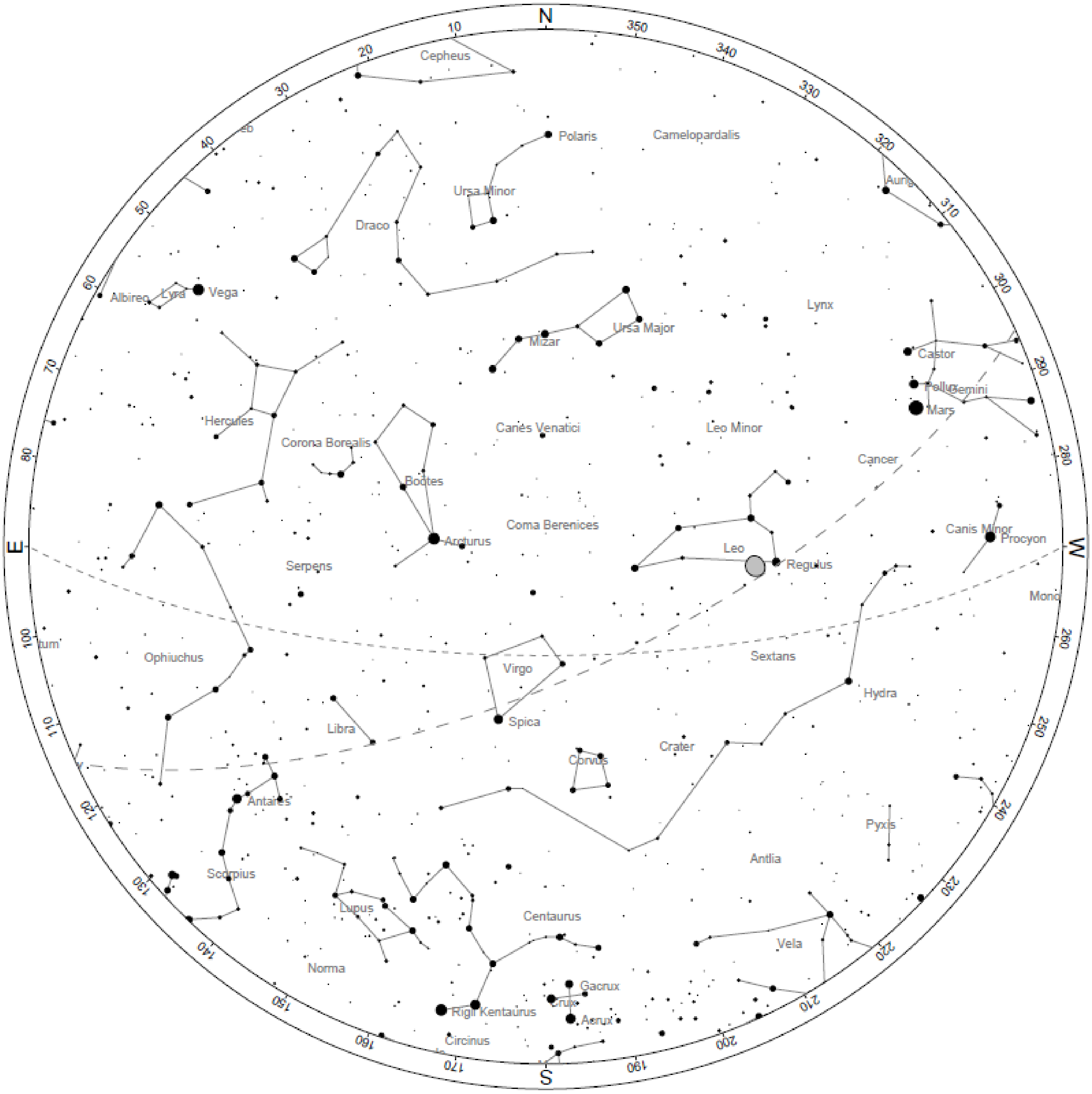
d)

- [0.5 marks] Temperature at radiation-matter equality 9277 K
- [0.5 marks] Energy of photons 0.80 eV (must be in eV)
- Age of Universe at z_rad is 6.73e4 years

e) Needs to be 9 marks

- Characteristic energy at freeze out 0.782 MeV
- N_n / N_p at freeze out = 0.191
- T at freeze out = 9.07e9 K
- z at freeze out = 3.33e9
- Age of universe at freeze out = 2.22 s OR age of universe at start of BBN = 285.58 s
- Time for neutron decay = 283 s
- Fraction of neutrons that do not decay = 72.5% and so N_n / N_p after neutron decay = 0.139
- Expression for Y_He is $\frac{4\left(\frac{N_n}{2}\right)}{N_n+N_p}$ or equivalent
- Calculate Y_He = 0.244

Q6 – Observational Astronomy



a)

- A = Vega
- B = Procyon
- C = Spica

b)

- Arcturus

c)

- Expect 19° (allow 17°-21°)

d)

- Gemini

e)

- Leo

f) [Any **three** from]

- Virgo
- Leo
- Canis Minor
- Ophiuchus
  - Also accept Serpens, Sextans, Hydra
  - For any others, use Stellarium and your discretion!

g)

- Star removed from Ursa Major
- Star added to Ursa Minor

h)

- M57 (accept Ring Nebula)
- 60°

i)

- Summer Triangle – RISING
- Winter Triangle – SETTING

j)

Qualitative answer

[1] for matching any of the following constellations (Gemini; Cancer; Sagittarius; Capricorn; Virgo) to its month (June OR July; July OR August; December OR January; January OR February; September OR October)

Then the candidate should count forward or backward months to find the Sun's position.

[1] for counting in the correct direction for the method used.

[1] for counting through 3 months.

[1] for final answer of **December** (from sun in Sagittarius or opposite Gemini) OR **January** (from sun in (late) Sagittarius opposite Gemini) but NOT January from sun in Capricorn opposite Cancer.

For example: The constellation on the meridian at this time is Virgo. Given it is roughly 6 o'clock, if we count three zodiacal constellations (one for each two hours) backwards to give us the constellation on the meridian at midnight, Gemini, which we can see is setting, as we would expect of the zodiacal constellation on the meridian at midnight, at 6 o'clock. The sun is therefore opposite Gemini in Sagittarius, and the month is therefore December.

Quantitative answer (much more accurate!):

Any three from, with ecf:

- [1] for finding UTC=00:15 given that UTC+5:30=5:45

So the approximate hour angle of the sun at Greenwich is 12h15m.

- [1] As Mumbai is east of Greenwich, we find the hour angle of the sun there by adding (24h)*(73^o/360^o) to get an hour angle of 17h07m.

Then, because Regulus is close to the celestial equator, we measure the distance between Regulus and the Meridian along the horizontal diameter of the star map. This distance is 0.22 of the whole diameter.

- [1] So the sidereal time is 10h08m + 0.22*12h=12h46m

As sidereal time = hour angle + right ascension,

- [1] The right ascension of the sun is 12h46m-17h07m=-4h21m

Then one for final answer:

Multiply by 365.25/24 to get 66 days before the vernal equinox, which is 20th March this year, to get 13th January (which is pretty close to the actual 18th January!). So **January**. [1]