International Astronomy and Astrophysics Competition
Qualification Round 2021

Problem A : Observing the Night Sky (5 Points)

(1) 2k - 4.5k (one half) (2) Ursa Major (3) Comet, eye visibility (4) Geminids
(5) Cassiopeia (6) Cygnus (7) Andromeda

Problem B : Shock Wave Escape (5 Points)

$vt = \frac{1}{2}at^2 + d_{AU} \implies a > \frac{v^2}{2d_{AU}} \approx 139\, m/s^2 \implies$ The crew manages to escape!

Problem C : Mysterious Planet (5 Points)

(a) $GmM/R^2 = F = mv^2/R \implies M = v^2\frac{R}{G} = \left(\frac{2\pi R}{T}\right)^2 \frac{R}{G} = \frac{4\pi^2 R^3}{T^2 G} \approx 1.03 \cdot 10^{26}\, kg$
(b) (*Planet*) Neptune, (*Moon*) Naiad

Problem D : Gravitational Constant (5 Points)

(a)

$$E = \int_{R+h}^{\infty} F\, ds = -GmM\frac{1}{R+h} = -\frac{4\pi R^3}{3}\rho \cdot Gm\frac{1}{R+h} = -\frac{4\pi G}{3}m\rho \cdot \frac{R^3}{R+h}$$

(b)

$$\frac{mv^2}{2} + E(0) = E(h) \;\Rightarrow\; \frac{mv^2}{2} = -\frac{4\pi G}{3}m\rho R^3\left(\frac{1}{R+h} - \frac{1}{R}\right) \;\Rightarrow\; G = \frac{3}{8\pi}\frac{v^2}{\rho R^2}\left(1 - \frac{R}{R+h}\right)^{-1}$$

(c) $G \approx 6.63 \cdot 10^{-11}$

Problem E : Pulsars (5 Points)

(individual written solution)